$$X \sim \mathcal{U}([-1,1])$$

$$f(x) = \begin{cases} \frac{1}{2} & -1 < x < 1 \\ 0 & \text{alt.} \end{cases}$$

$$X \sim \mathcal{U}([a,b]) \qquad f(x) = \begin{cases} \frac{1}{b-a} & a<x<b \\ 0 & \text{---} \end{cases}$$

Y ind. da X $\boxed{P(Y=1) = P(Y=-1) = \frac{1}{2}}$

$U = XY$ ha la stessa legge di X

$$P(U \le t) = P(X \le t)$$

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$\boxed{Z = X + U}$ non è una v.a. continua

$\underbrace{P(Z=0)} = P(X+U=0) =$

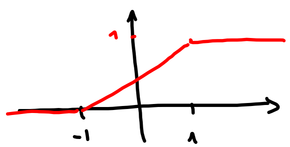
$= P(\underbrace{X(1+Y)} = 0) = P(1+Y=0)$

$= P(Y=-1) = \underbrace{\frac{1}{2}}$

(c) Calc. la f.d.r. di $Z = X(1+Y)$

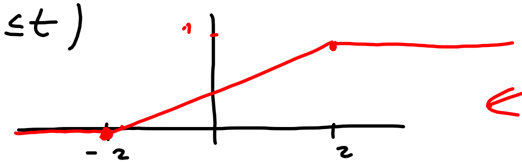
$$P(Z \le t) = P(X(1+Y) \le t) =$$
$$= P(X(1+Y) \le t, Y = 1) +$$
$$+ P(X(1+Y) \le t, Y = -1) =$$
$$= P(\underline{2X \le t}, \underline{Y=1}) + P(0 \le t, Y=-1)$$
$$= P(2X \le t)\, P(Y=1) + \underline{P(0 \le t)}\, P(Y=-1)$$
$$= \frac{1}{2} P\left(X \le \frac{t}{2}\right) + \frac{1}{2} P(0 \le t)$$

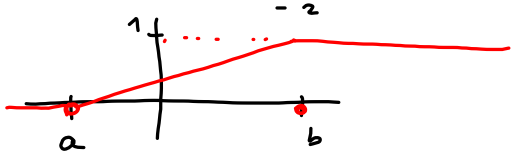
$$P(X \le u) = \begin{cases} 0 & u < -1 \\ \frac{u+1}{2} & -1 \le u \le 1 \\ 1 & u > 1 \end{cases}$$



$$P\left(X \le \frac{t}{2}\right) = \begin{cases} 0 & t < -2 \\ \frac{\frac{t}{2}+1}{2} = \frac{t+2}{4} & -2 \le t \le 2 \\ 1 & t > 2 \end{cases}$$

$$P(2X \le t)$$





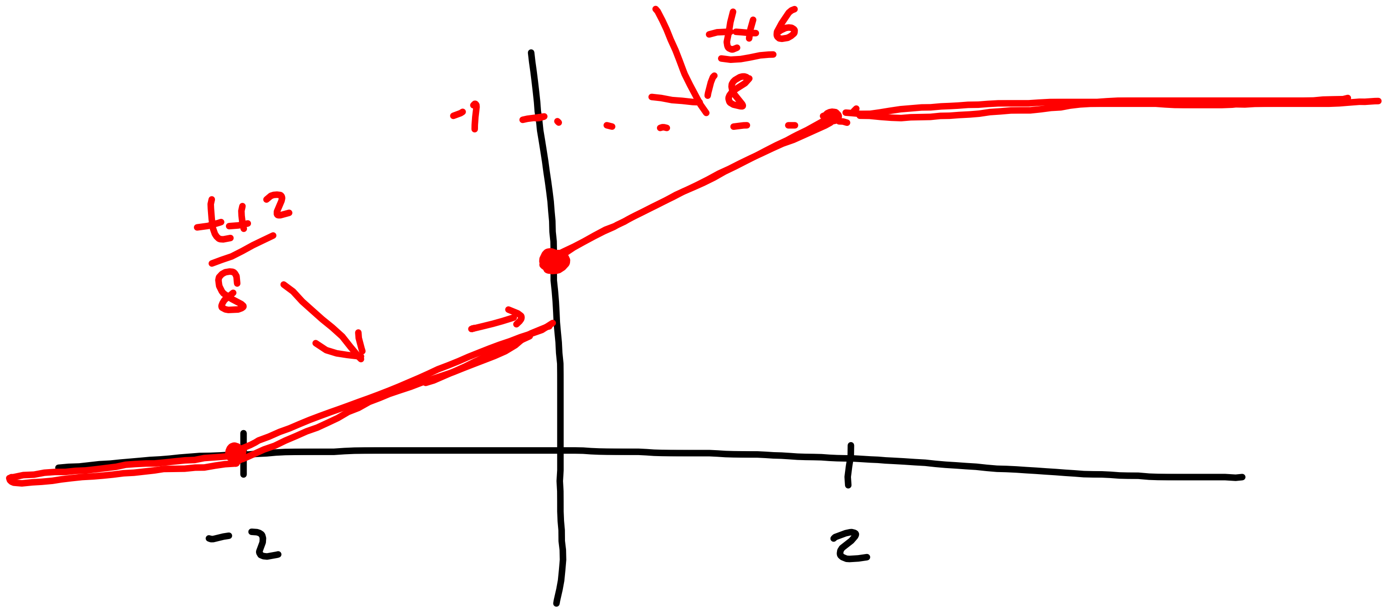
$2X \sim \mathcal{U}(-2, 2)$

$$\frac{1}{2} P\left(X \le \frac{t}{2}\right) + \frac{1}{2} P(0 \le t) =$$

$$= \frac{1}{2} \begin{cases} 0 & \boxed{t < -2} \\ \frac{t+2}{4} & -2 \le t \le 2 \\ 1 & t > 2 \end{cases} + \frac{1}{2} \begin{cases} 0 & t < 0 \\ 1 & t \ge 0 \end{cases}$$

	-2		0		2	
0		$\frac{t+2}{4} + 0$		$\frac{t+2}{4} + 1$		$1+1$
0		$\frac{t+2}{8}$		$\frac{t+6}{8}$		1

fare il grafico $\quad P(Z = 0)$



$$P(Z=0)=\frac{6}{8}-\frac{2}{8}=\frac{1}{2}$$

$L \to \frac{1}{3}$

$B \to \frac{2}{3}$

- Se il segnale è L esso impiega un tempo aleatorio $\sim \mathcal{E}(2)$
- Se il segnale è B, il tempo $\sim \mathcal{E}(1)$

Z = tempo impiegato dal segnale per giungere a destinazione

(a) Calcolare $P(Z \le t \mid B)$ $P(Z \le t \mid L)$

dove $B = \{$il segnale è $B\}$ $L = \{\dots$ è $L\}$

(b) Calcolare la f.d.r. di Z e $E[Z]$

(a) $P(\underline{Z \le t} \mid B) = \begin{cases} 0 & t < 0 \\ 1 - e^{-t} & t \ge 0 \end{cases}$

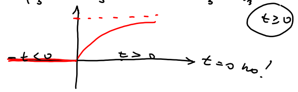
$$P(Z \le t \mid L) = \begin{cases} 0 & t < 0 \\ 1 - e^{-2t} & t \ge 0 \end{cases}$$

$$P(\underbrace{Z \le t}_{A}) = ?$$

$$P(A) = P(A \mid B)P(B) + P(A \mid L)P(L)$$

$$= \frac{2}{3}\begin{cases} 0 & t < 0 \\ 1 - e^{-t} & t \ge 0 \end{cases} + \frac{1}{3}\begin{cases} 0 & t < 0 \\ 1 - e^{-2t} & t \ge 0 \end{cases} =$$

$$= \begin{cases} 0 & t < 0 \\ \frac{2}{3}(1 - e^{-t}) + \frac{1}{3}(1 - e^{-2t}) = 1 - \frac{2}{3}e^{-t} - \frac{1}{3}e^{-2t} & t \ge 0 \end{cases}$$



$F(t)$ f

$$\underline{F(t)} = \int_{-\infty}^{t} \underline{f}(x)\,dx = \int_{-\infty}^{a} f(x)\,dx + \int_{a}^{t} f(x)\,dx$$

$$F(t) = c + \int_{a}^{t} f(x)\,dx$$

$$F'(t) = f(t)$$

Teor. fond. del calcolo

Sia f una funzione continua su $[a,b]$. Posto

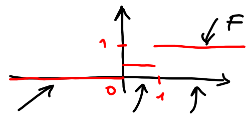
$$F(t) = \int_{a}^{t} f(x)\,dx$$

allora F è derivabile in (a,b) e

$$F'(t) = f(t) \quad \forall t \in (a,b)$$

$F \qquad F' = f$

$$F(t) = \int_{-\infty}^{t} f(x)\,dx \qquad \forall t$$



$f(x) = 0 \quad \forall x$

$$F(t) = P(Z \le t) = \begin{cases} 0 & t < 0 \\ 1 - \frac{2}{3}e^{-t} - \frac{1}{3}e^{-2t} & t \ge 0 \end{cases}$$

$$f(t) = \begin{cases} 0 & t \le 0 \\ \underbrace{\frac{2}{3}e^{-t} + \frac{2}{3}e^{-2t}} & t > 0 \end{cases}$$

$$\int_{-\infty}^{t} f(x)\,dx \qquad \forall t$$

$$E[Z] = \int_{-\infty}^{+\infty} t\, f(t)\,dt =$$

$$= \int_{0}^{+\infty} t\left(\frac{2}{3}e^{-t} + \frac{2}{3}e^{-2t}\right)dt =$$

$$= \frac{2}{3}\underbrace{\int_{0}^{+\infty} t e^{-t}\,dt} + \frac{1}{3}\underbrace{\int_{0}^{+\infty} 2t e^{-2t}\,dt}$$

$$= \frac{2}{3}\cdot 1 + \frac{1}{3}\cdot\frac{1}{2} = \frac{2}{3} + \frac{1}{6}$$

Se $X \sim \mathcal{E}(\lambda)$

$$E[X] = \int_{0}^{+\infty} t\,\lambda e^{-\lambda t}\,dt = \frac{1}{\lambda}$$

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Sia $X \sim \mathcal{E}(\lambda)$

(a) Calc. la fdr di $Y = \frac{1}{X}$

(b) Calc. la densità di Y

(c) calc. $E[Y]$.

(a) $P(Y \le t) = P\left(\frac{1}{X} \le t\right) = \underline{P(1 \le X}t)$

$$P(X \le t) = \begin{cases} 0 & t \le 0 \\ 1 - e^{-\lambda t} & t > 0 \end{cases} \qquad P(X \le 0)$$

$$X \sim f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & x \le 0 \end{cases}$$

$$P\left(\frac{1}{X} \le t\right) = P\left(\frac{1}{X} \le t, X > 0\right)$$

$$+ P\left(\cancel{\frac{1}{X} \le t, X < 0}\right) =$$

$$= P(1 \le Xt, X > 0) + P(1 \ge Xt, X < 0)$$

$\underline{P(Y \le t)} = P\left(\frac{1}{X} \le t\right) = P(1 \le Xt)$

$= P\left(\frac{1}{t} \le X\right) \qquad t > 0$

Se $t \le 0$

$P\left(\frac{1}{X} \le t\right)$

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$X \sim \mathcal{E}(\lambda) \qquad Y = \frac{1}{X}$

Dato che $P(X > 0) = 1$, anche $P(\frac{1}{X} > 0)$

Dunque $P(\frac{1}{X} \leq t) = 0$ se $\underline{t \leq 0}$

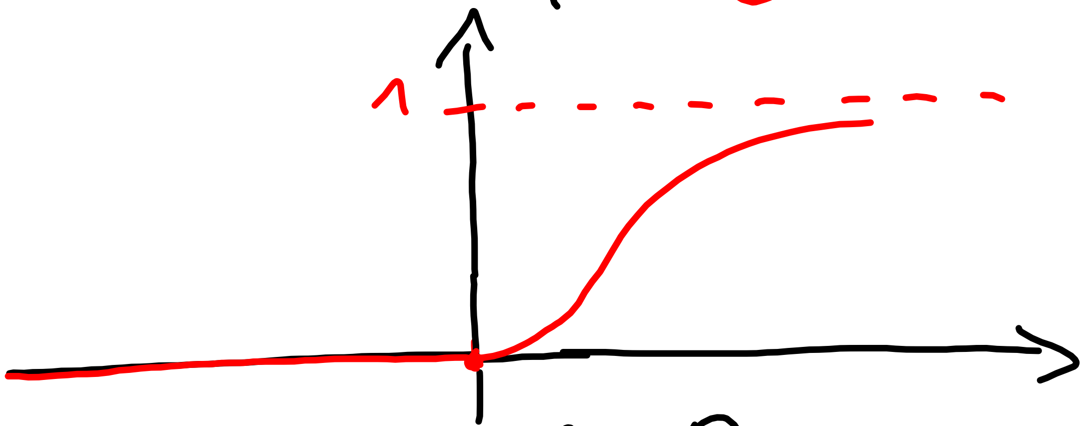
$\rightarrow$ Se $t > 0$

$$P\left(\frac{1}{X} \leq t\right) = P(1 \leq tX) = P\left(X \geq \frac{1}{t}\right) =$$

$$= 1 - P\left(X < \frac{1}{t}\right) = 1 - P\left(X \leq \frac{1}{t}\right) =$$

$$= 1 - \begin{cases} \cancel{0} & \cancel{\frac{1}{t}} \leq 0 \\ 1 - e^{-\lambda \frac{1}{t}} & \frac{1}{t} > 0 \end{cases} = e^{-\frac{\lambda}{t}}$$

$$P(Y \leq t) = \begin{cases} 0 & t \leq 0 \\ e^{-\frac{\lambda}{t}} & t \geq 0 \end{cases}$$



$$f(x) = \begin{cases} 0 & x \leq 0 \\ e^{-\frac{\lambda}{x}} \dfrac{\lambda}{x^2} & x > 0 \end{cases}$$

$$\lim_{t \to 0^+} \frac{\cancel{\lambda}}{t^2} e^{-\frac{\cancel{\lambda}}{t}} = \lim_{x \to +\infty} x^2 e^{-x} = 0$$

$$E[Y] = \int_{-\infty}^{+\infty} y\, f(y)\, dy =$$

$$= \int_0^{+\infty} y\, \frac{\lambda}{y^2} e^{-\frac{\lambda}{y}}\, dy =$$

$$= \int_0^{+\infty} \frac{\lambda}{y} e^{-\frac{\lambda}{y}}\, dy$$

$$\frac{1}{y} = u \qquad y = \frac{1}{u} \qquad dy = -\frac{1}{u^2}\, du$$

$$= -\int_{+\infty}^{0} \lambda u\, e^{-\lambda u} \frac{1}{u^2}\, du =$$

$$= \int_0^{+\infty} \frac{\lambda}{u} e^{-\lambda u}\, du = +\infty$$

$$\Gamma(\alpha) = \int_0^{+\infty} u^{\alpha - 1} e^{-\lambda u}\, du \qquad \alpha > 0$$

$$X \sim \mathcal{E}(\lambda) \qquad Y = \frac{1}{X} = \varphi(X)$$

$$E[Y] = E[\varphi(X)] =$$

$$= \int_{-\infty}^{+\infty} \varphi(x) f(x)\, dx = \int_{0}^{+\infty} \frac{1}{x} \lambda e^{-\lambda x} dx$$

dove f è la densità di X

Se $X \sim \mathcal{U}([a,b])$, allora la v.a. $Y = mX + n \sim \mathcal{U}([ma+n, mb+n])$

$2X$ $\qquad$ $\mathcal{U}([2a,2b])$